

# Analysis and use of RSA keypair generation bias

Learning from the discovery, disclosure, and mitigation of vulnerable RSA key generation on smartcards

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Credits: Petr Švenda



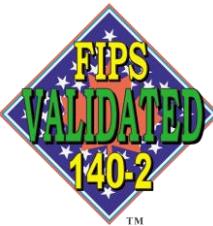
**CRCS**

Centre for Research on  
Cryptography and Security

[www.fi.muni.cz/crocs](http://www.fi.muni.cz/crocs)

# About CROCS lab

- Masaryk University, Brno
- Smartcards analysis from 2002
- Cryptographic smartcards are pervasive (SIM, EMV, eID, tokens...)
- Yet smartcard industry is very closed
  - NDA just to see detailed specifications, proprietary APIs, no design details...
- Security certifications performed by testing labs (FIPS, CC)
  - But details are not public
- Idea in 2014: Infer details using keys similarity to open-source libraries



## RSA primer – what does it mean and why should I care?

- RSA is widely used public-key cryptosystem (1977)
- Used for digital signatures (mail, software distribution, contracts...)
- Used for key exchange (HTTPS/TLS, PGP...)
- Private part: random primes **P** and **Q**, private exponent **d**
- Public part: modulus **N**, public exponent **e** (often 65,537)

$$\textcolor{red}{P} \times \textcolor{red}{Q} = \textcolor{green}{N}$$



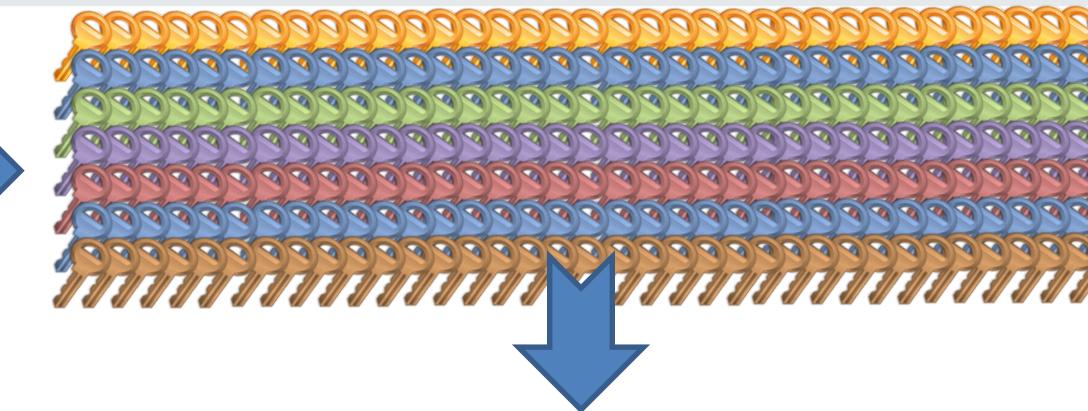
Factorization attack: compute primes **P** and **Q** from the knowledge of **N**

- Problem: How to generate a large prime (1024- or 2048-bit length)?

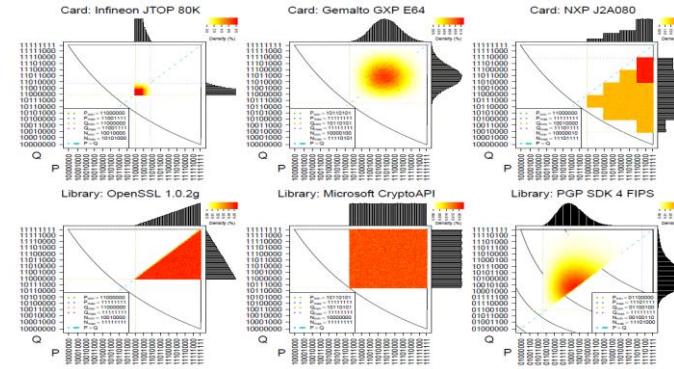


# 60+ million fresh RSA keypairs

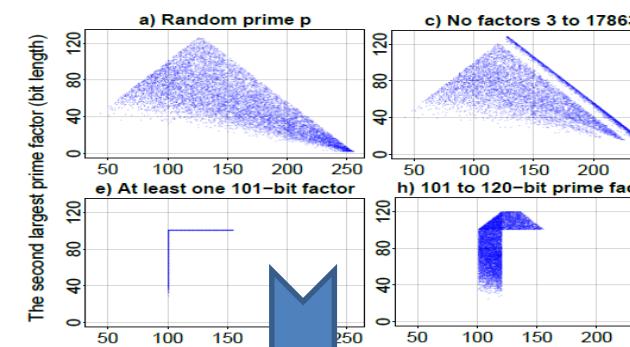
22 sw. libraries  
16 smart cards



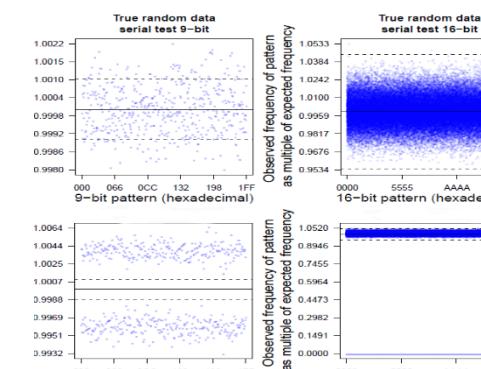
Distribution of primes (MSB)



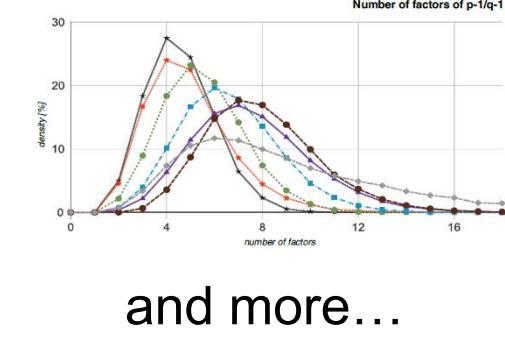
Large factors of  $p-1$  /  $p+1$



Bit stream statistics



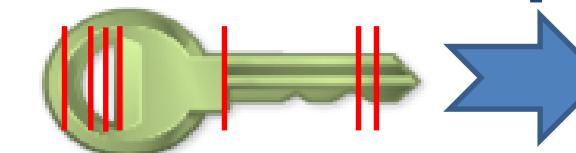
Number of factors



and more...

## 7 implementation choices observable in public keys

(biased bits of public modulus, “mask”)



**OpenSSL**

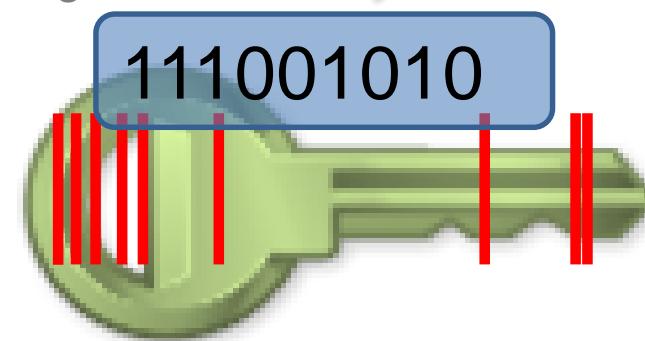
# Input key

-----BEGIN CERTIFICATE-----



MIIG9zCCBd+gAwIBAgIJOR2wFUwc20wDQYJKoZI  
hvcNAQELBQAwSTELMAkGA1UEBhMCVVMxEzAR  
BgNVBAcTCkdyb2ds7mBjcmMxJTAjBgNVBAMTHE  
dvb2ds7mBjcm5ldCBXRob3JpdHkgRzlwHhc  
NMTYNTA5NzA5MDIwOTI4MDgwMzA  
wk2zl...an...R0QD9zPkrrEp4miQ9aVgC6k7i  
bLukl4c...e0kCQr8kNUBhH25DS6HpekTmO1s  
9q81KbtS2E7+4Q/57xdgħBLiaTEv7O7+gskLQ/qJa  
TouwiDPM6SHIVU6X2Ca1INKg2wbx8h2Q63SDIwFJ  
52HsNACIKp4ADvjvvlmYoWVitcLihpXogOAzbLz3Hls  
6Jk=

-----END CERTIFICATE-----



# Precomputed matrix

Mask value	Group I	Group II	...	Group XII	Group XIII
000000000	0.124	0.347		0.105	0.012
000000001	0.004	0.038		0.236	0.454
000000011	0.046	0.002		0.447	0.112
...					
111111110	0.394	0.044		0.320	0.002
111111111	0.046	0.347		0.015	0.312



# Classification

44%  OpenSSL's group

11%  PolarSSL's group

9%  PGP®'s group

...

1. Insert your key/s

## Morphology of your RSA public keys

Insert or drag & drop public RSA key/s

-----BEGIN CERTIFICATE-----

MIIGpzCCBY+gAwIBAgIQfa+N  
hDE7MDkGA1UECwwyZ2VuZG  
TC9UTFMgc2Nhbm5pbmcxHzA  
JDAiBgNVBAMMG2F2YXN0ISI  
MDAwMDBaFw0xNzA0MjMyM  
IFZhbgIkYXRIZDEhMB8GA1UE  
IgYDVQ0NDExzhmkxOTk0NiIu

2. Basic result

3. Details

1. Insert your key/s

2. Basic result

3. Details

## Morphology of your RSA public keys

### RSA public keys

We found 3 RSA public keys.

### Most probable source/s

The most probable source is OpenSSL 1.0.2g.

### Results accuracy

We are 82.27 % sure, that your keys were generated by this source.

### Negative results

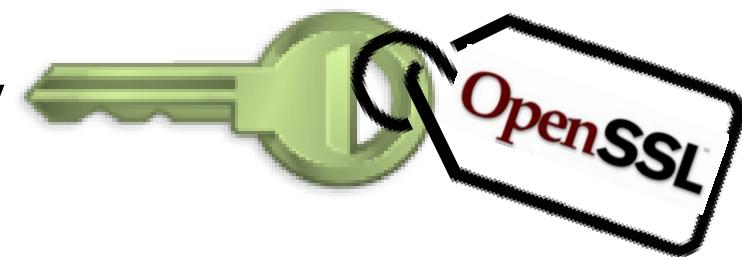
Your key could be generated by all widely used software libraries.

◀ Insert more keys

Detailed results ➔

# Impact (of the possibility) of public key classification

- Information leakage vulnerability



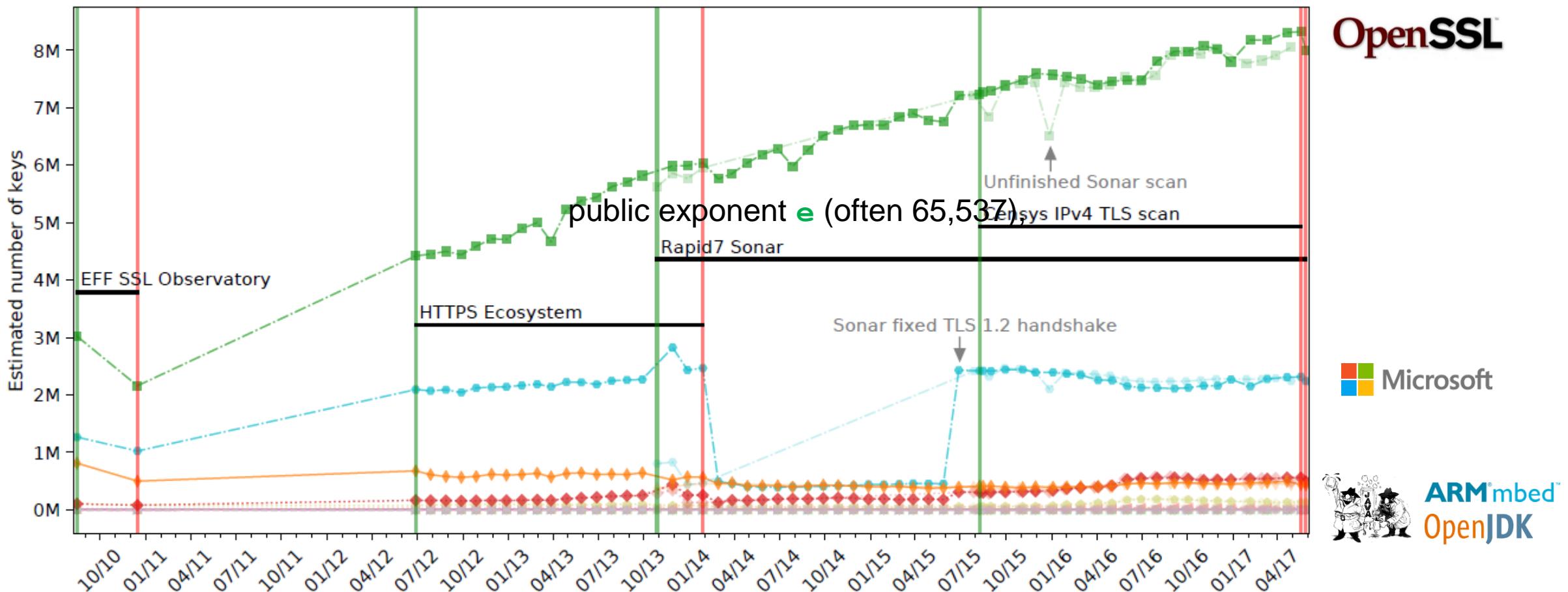
# Impact (of the possibility) of public key classification

- Information leakage vulnerability
- Statistics: current library usage trends



RSA key classification  
(USENIXSec'16, ACSAC'17)

# Internet-wide TLS IPv4 scan



# Impact (of the possibility) of public key classification

- Information leakage vulnerability
- Statistics: current library usage trends
- Audit: identify source libs in target organization



RSA key classification  
(USENIXSec'16, ACSAC'17)

EE eID injected keys  
(Arnis Paršovs, 05/2018)

# Problem reported from Estonia (17.5.2018)

- Estonian eIDs generate private key always on chip (by design)
  - Some keys found to be injected from outside
- Found by observed discrepancy in public key properties (MSB)
  - Expected:  $\text{MSB} \in \{144, 145, \dots, 167\}$
  - Observed: outside

The ID-card maker has violated the most important security principle and 12,500 cards need to be replaced by people.

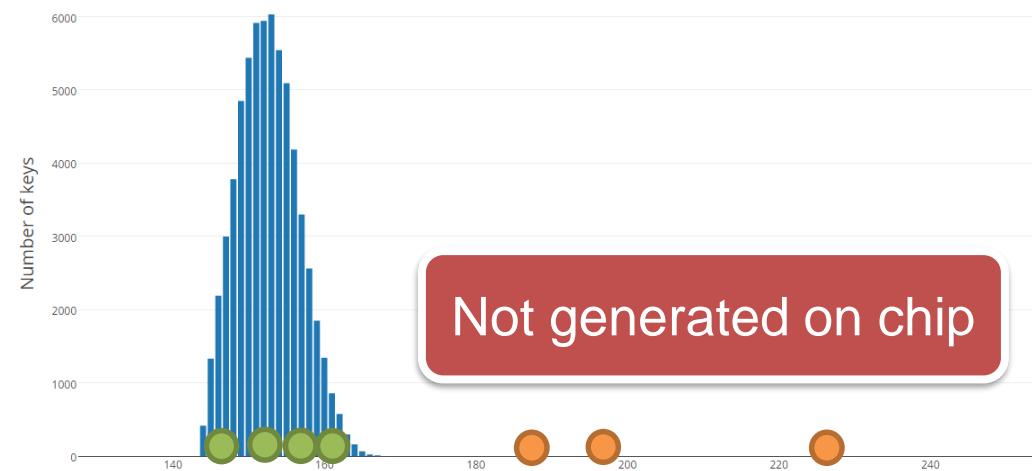


Hans Löug

□ 05/27/2018 at 13:58



1 like Meeldib 32



# Impact (of the possibility) of public key classification

- Information leakage vulnerability
- Statistics: current library usage trends
- Audit: identify source libs in target organization
- Forensics: source lib/device of weak keys
- Quick search for other keys from vulnerable library



RSA key classification  
(USENIXSec'16, ACSAC'17)

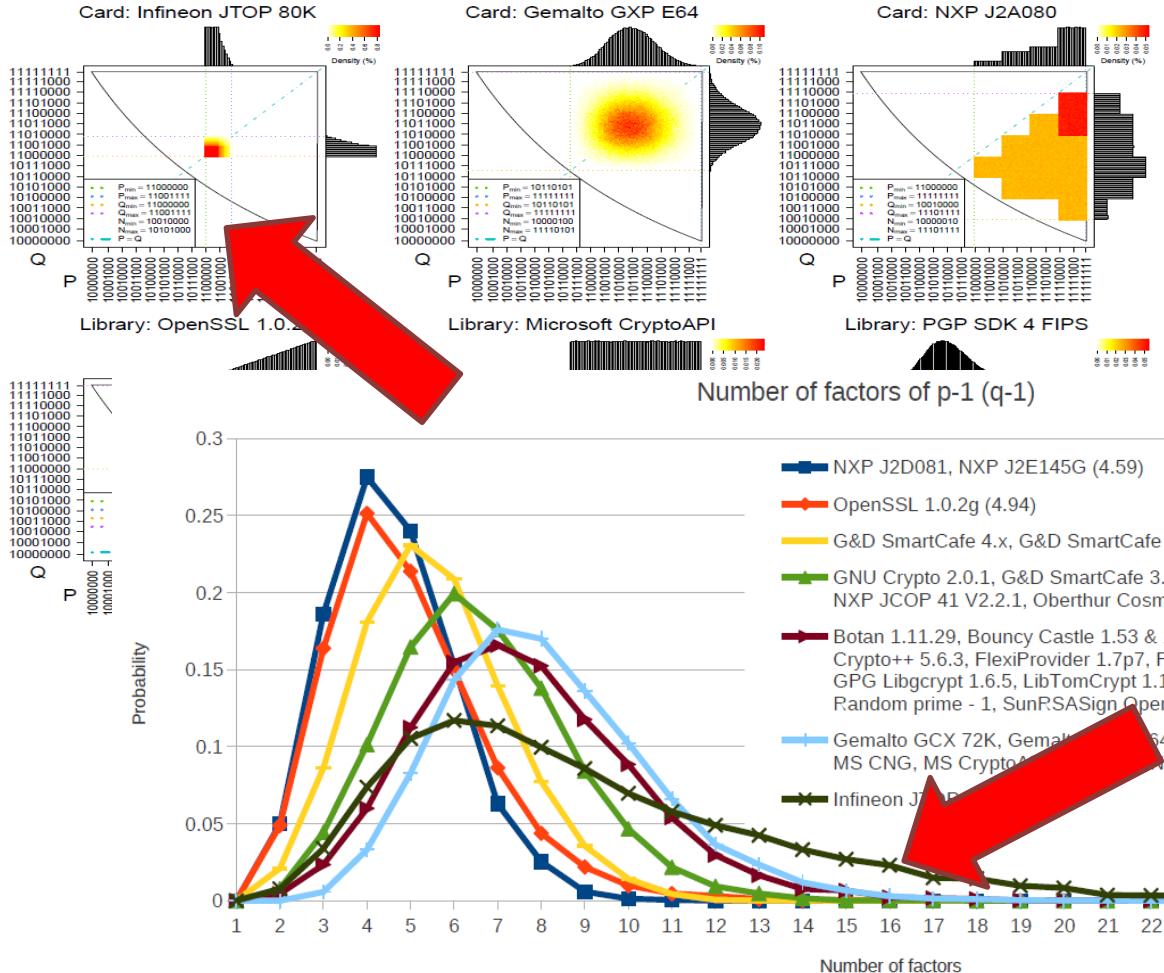
EE eID injected keys  
(Arnis Paršovs, 05/2018)

Source of factorable TLS  
keys (I. Mironov, 2012)

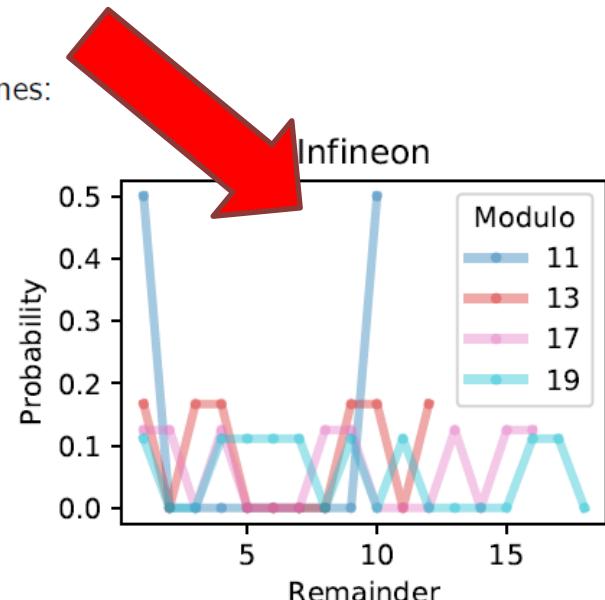
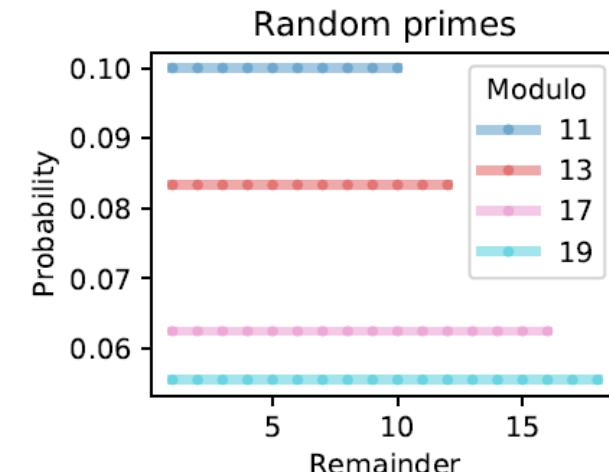
ROCA vulnerability  
(ACM CCS'17)

# ROCA vulnerability

# But we were unaware on another issue that time



Distribution of RSA keys modulo small primes:



**Prime<sub>expected</sub> = random**

**Prime<sub>Infineon</sub> =  $k * M + 65537^a \bmod M$**



# Prime generation

**slow !** - primality tests (modular exponentiation)

## Algorithms

1. Random sampling – **generate & test, generate & test, ...**
  - Many iterations – random number has typically small divisors
  - 50% that 2 is divisor, 33% that 3 is divisor ...
2. Incremental search – **generate & test, increment & test, increment...**
  - skip numbers with small prime factors
  - similar methods **Joye & Pailier algorithm** , “Fast Prime” algorithm (Infineon)

# Structure of Infineon primes

$$\text{prime} = k \cdot M + 65537^a \bmod M, \quad M = 2 * 3 * 5 * 7 \dots$$

- Entropy loss in prime:



Consequences:

- Strong fingerprint of RSA keys
- Practical factorization of RSA keys is possible

## Detection of vulnerable keys

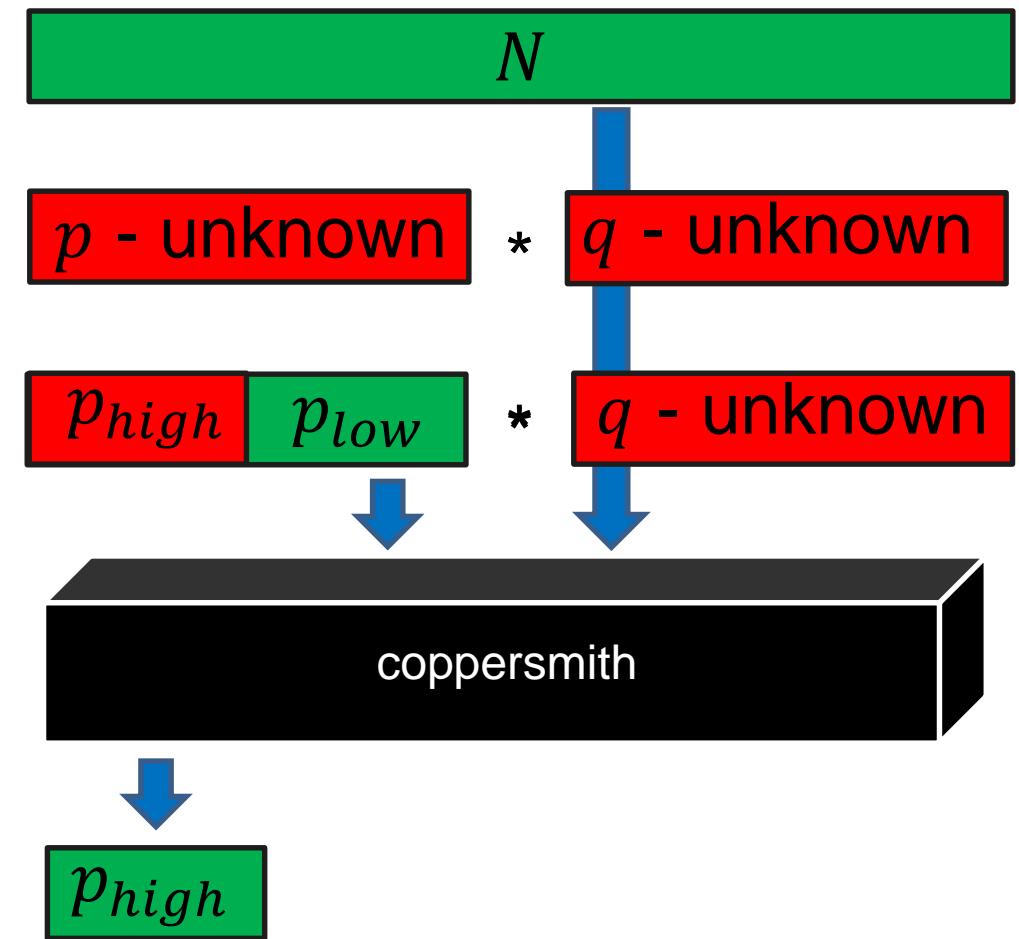
- Based on public modulus  $N$
- 1. Vulnerable if  $c$  exists:
- 2. Equivalent to  $c_i$  exist for **all**  $p_i \mid M$ :
  - small  $p_i \Rightarrow$  very fast - microseconds
- Errors:
  - False negatives - all Infineon primes have the specific form
  - False positives - negligible probability ( $Pr < 2^{-150}$  )

$$N \equiv 65537^c \bmod M$$

$$N \equiv 65537^{c_i} \bmod p_i$$

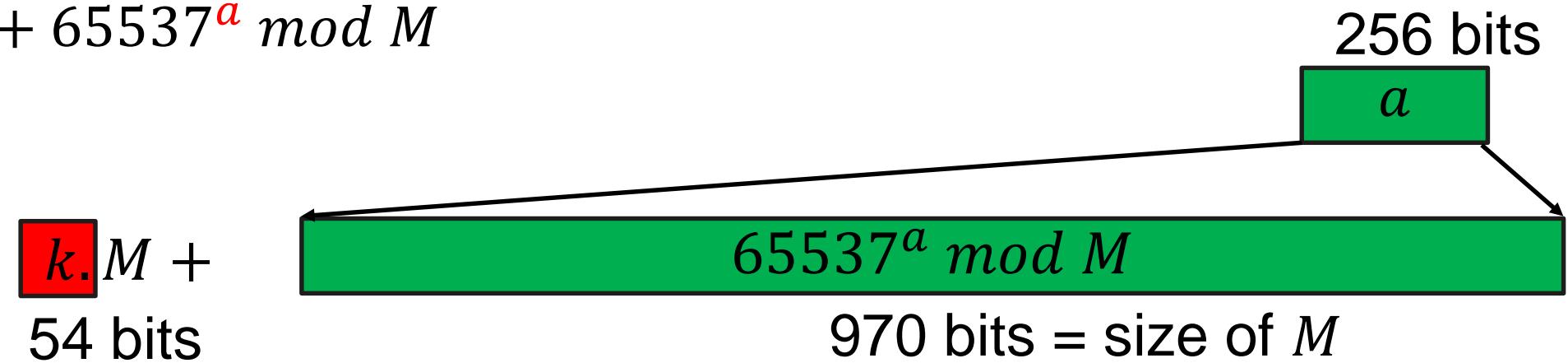
# Coppersmith's attack as a black box

1. Modulus  $N$
2. Unknown factors  $p, q$
3. Partial knowledge of prime  
**(at least  $\frac{1}{2}$  of bits of  $p$ )**
4. Apply Coppersmith's algorithm



## Naïve algorithm (RSA -2048)

- $p = k \cdot M + 65537^a \text{ mod } M$
- Guess  $a$

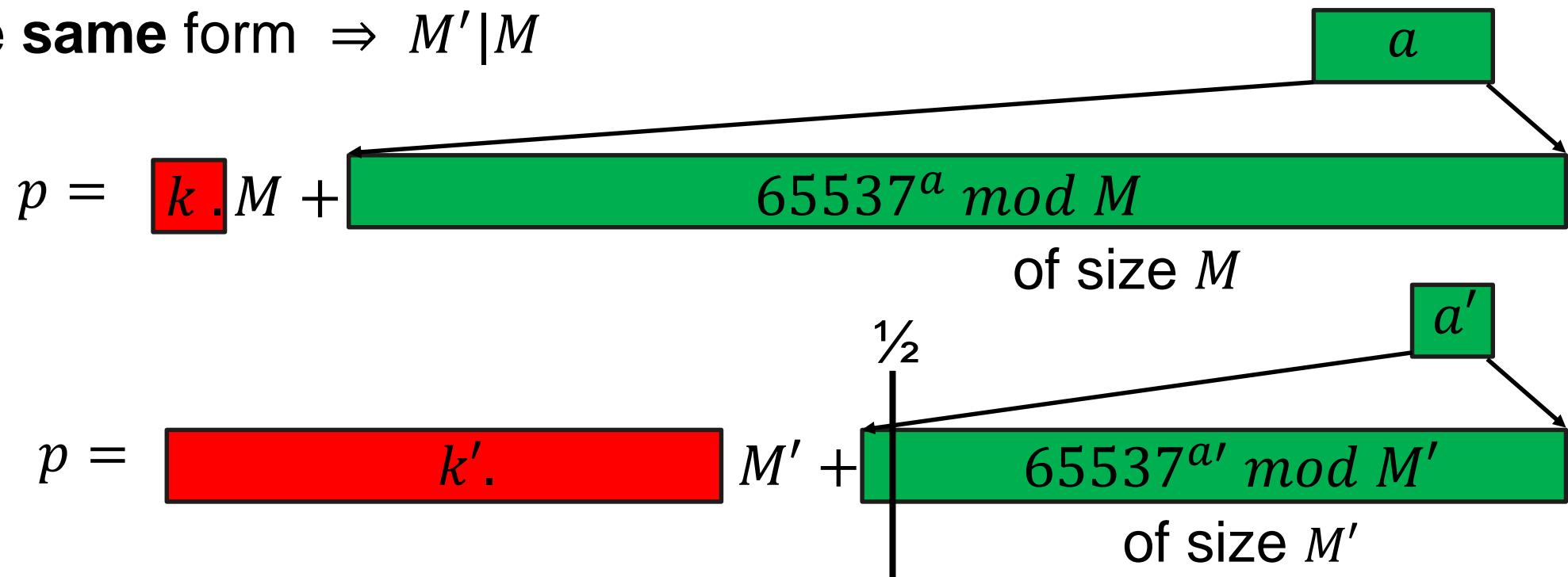


- compute  $k$  using Coppersmith's alg.  
(requires  $\frac{1}{2}$  of known bits – much more than that – large  $M$ )
- **Infeasible** – large  $a$

# How to make attack practical ?

Idea:  $\frac{1}{2}$  known (= size of  $M$ ) bits of  $p$  is sufficient

- smaller  $M' \Rightarrow$  smaller (or equal)  $a'$
- $p$  of the **same** form  $\Rightarrow M'|M$



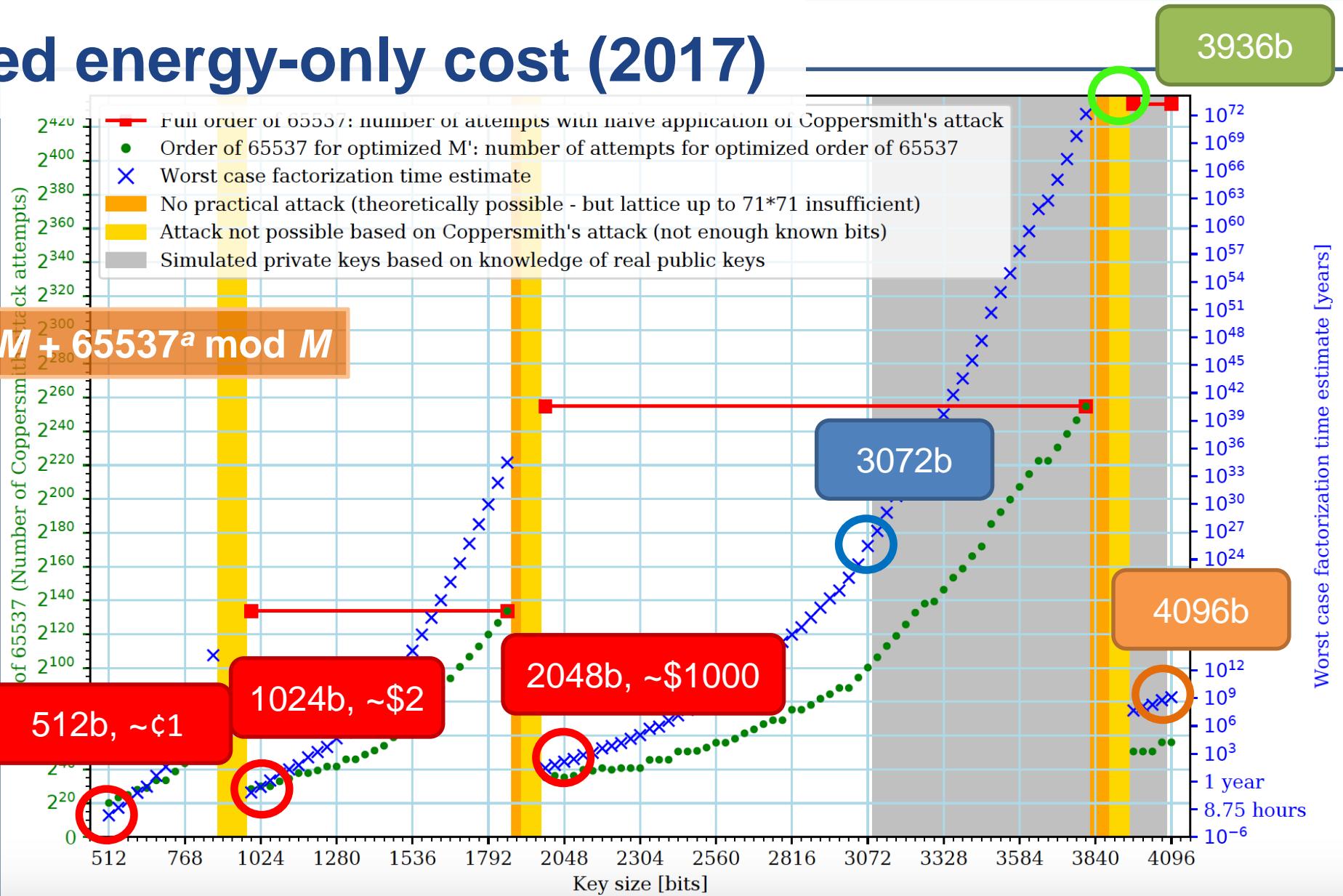
# ROCA in general

# Algorithmic flaw in Infineon’s RSALib (CVE-2017-15361)

- All keys generated by vulnerable Infineon library are affected
- Practical factorization of common lengths 512/1024/2048b (+ others)
  - Randomly selected 512 and 1024b keys factorized (Masaryk University)
  - Randomly selected 2048b (Estonian RIA, April 2018, “several thousands euro”)
- All public keys have unique “fingerprint” (easy to scan for)
  - Tool for detection, <https://github.com/crocs-muni/roca/>
- Tool for factorization (made public by Lange&Bernstein, 5<sup>th</sup> Nov)
  - Random 2048b key: 6442450944000000 vCPU years
  - Infineon 2048b key: **140** vCPU years

Attack is perfectly parallelizable  
1000 cores => 1000x speedup

# Estimated energy-only cost (2017)



## What is the cost of an attack on RSA 2048b (year after)?

- Our paper (2017): \$20,000 average price on Amazon AWS
  - Estimate: energy-only price is likely around \$1000
- **Lange, Bernstein (2017) – 25% faster attack (LLL chaining)**
  - Found in **3 days** and without an access to our paper!
- Estonian RIA (04/2018): “several thousand euros” energy price
- Our work (WAC 2018): algorithmic improvement, 2x faster
- Implementation speedups by graphic cards, FPGA...
  - Not (publicly) tested (typical speed-up factor 3-10x)

# Responsible disclosure I.

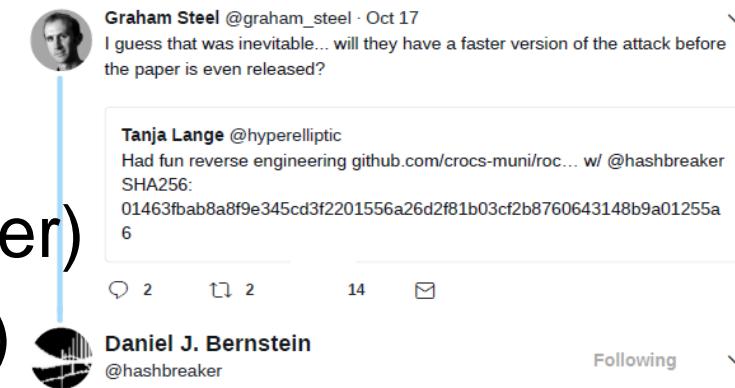
## Recipients

Austria, Belgium, Bulgaria, Cyprus, Czech Republic, Denmark, Estonia, Finland, France, Greece, Hungary, Ireland, Italy, Latvia, Lithuania, Luxembourg, Malta, Netherlands, Poland, Romania, Slovakia, Slovenia, Spain, Sweden, United Kingdom, Liechtenstein, Iceland,

- (NIST responsible disclosure guidelines followed)
- End of January 2017: Proof of Concept attack (1024b keys factorized)
- Feb 1<sup>st</sup>: Infineon notified (email to contact at crypto group)
- Mid May: First Infineon's customers contact us back for verification
- Jun 20<sup>th</sup>: Incident report ID 163484, Austria eHealth certs revoked
  - eIDAS regulation and Article 19
  - Countries around Europe should have been notified
  - BUT: unspecific third party failure, concrete vendor named (but not Infineon)

## Responsible disclosure II.

- Last week Aug: vulnerable new EE certs detected (LDAP scan)
- Aug 30<sup>th</sup>: EE CERT formally contacted by us
- Sept 5<sup>th</sup>: Estonia publicly announced eID issue
- Oct 10<sup>th</sup>: Microsoft Patch Tuesday (TPMs, Bitlocker)
- Oct 16<sup>th</sup>: Public disclosure (coincide with KRACK)
  - Impact announced by us, detection tool released
- Oct 23<sup>rd</sup>: Lange& Bernstein announced faster attack
- Vulnerable devices from year 2007 found (Gemalto IDPrime .NET)
- Oct 30<sup>th</sup>: Full paper with details published (ACM CCS)
- 2/3.11. Slovakia/Estonia revokes 300k/760k certificates (10M in Spain)



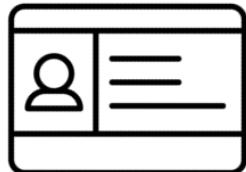
Replies to @graham\_steele  
Yup. Our 2048bit attack using @sagemath is now 5-25% faster than ROCA blog.  
31d6a53a3b6362248ac10de4a8108df3c839a  
7193a96d0991c6675990599d917

M. Nemeč, M. Sys, P. Svenda, D. Klinec, V. Matyáš: The Return of Coppersmith's Attack..., ACM CCS 2017

## The usage domains affected by the vulnerable library

Austria, Estonia,  
Slovakia, Spain...

Identity documents  
(eID, eHealth cards)



Trusted Platform Modules  
(Data encryption, Platform integrity)



25-30% TPMs worldwide,  
BitLocker, ChromeOS...  
Firmware update available

Software signing



Commit signing,  
Application signing  
GitHub, Maven...

Authentication tokens



Gemalto .NET  
Yubikey 4...

RSA Library



Affected chip

Secure browsing  
(TLS/HTTPS\*)



Very few keys, but all tied  
to SCADA management

- Impact on document signatures
  - Limited by time stamps + revocation
- Impact on encrypted data
  - Still relevant (need perfect forward secrecy)

\* only a small number of vulnerable keys found

# What were impacted parties typically struggling with?

- Is this attack really practical or “just” theoretical?
- How to mitigate / update already distributed cards/tokens?
  - Estonia remote update of eIDs JavaCard application (RSA → ECC)
  - Slovakia RSA 2048b → RSA 3072b
  - Yubico: free token replacement
  - Gemalto .NET auth cards?
- Is migration to 3072b safe? (BSI says ok)
- What is actually certified? (TRNG→primes→key→use of private key)
- How to revoke large number of certificates?

## Conclusions

- Certified != Secure
- Follow standards and methods!
- Every leak is problematic but
  - $\frac{1}{2}$  leaked bits (Public key crypto) => complete break
- Secret design => delayed flaw discovery => higher impact
- Be prepared to revoke, patch and update everything



## Are there any positives from ROCA vulnerability?

- Critical, long-present vulnerability mitigated
  - Vulnerable keys testing incorporated in administrators tools (Let's Encrypt...)
- Speed-up transition to ECC or at least longer RSA keys
- Changes to standard - verifiable RSA keypair generation from seed
- Changes to certification process - more scrutiny for key generation
- Sparked discussion about more efficient information sharing (eIDAS)
- ...

Another argument for more openness  
and certification transparency?



# Minerva vulnerability (10/2019)

<https://minerva.crocs.fi.muni.cz/>

- Discovered by ECTester (<https://github.com/crocs-muni/ECTester>)
- Athena IDProtect smartcard (EAL 4+)
  - FIPS140-2 #1711, ANSSI-CC-2012/23
  - Inside Secure AT90SC28872 Microcontroller
  - (possibly also SafeNet eToken 4300...)
- Libgcrypt, wolfSSL, MatrixSSL, Crypto++
- SunEC/OpenJDK/Oracle JDK
- Small time difference leaking few top bits of nonce
- Enough to compute whole EC private key in 20-30 min
  - ~thousands of signatures

